Mobile Service Pricing Strategy Analysis from Social Tie and Network Effects

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Abstract: There is a big tendency in the world telecom market that mobile carriers rolled out the strategy to offer social-network-based service plan, such as family cell phone plan in U.S., Italy and Spain, group free calling service in China and France, and family and friend calling plan in Fiji. Although the plan in different countries varies is various ways, telecom carriers all offer these different size group of person with discounted prices and the discount only is restricted to communication within telecom carriers' network. In the paper, we look at both network effects and social network effects in one part nonlinear pricing discrimination strategy adopted telecom carriers. I assume that the value of telecommunication among a pair of the communicators depends on network size and the social tie between the communicators in the network. I will address several questions surrounding this telecom pricing strategy: why company adopts this pricing strategy, how it affects consumer demand as well as value, how it affects firm's profit, and how to design this strategy. We found that telecom operators need to offer various service tariff plan to target different types of consumers identified by strong social tie or weak social tie in their networks. The purpose of this paper is to model personal social networks and social tie with consideration of network effects, and then to analyze the current popular telecom pricing strategy from network effects and social network perspective.

1. INTRODUCTION

There is a significant tendency of widely adopted social network based telecom pricing strategy in the world telecom market today. Social network based telecom pricing strategy refers to price discrimination strategy based on subscriber's particular social tie or networks such as working partners, families, and friends. We are observing this popular strategy being used in different markets with different package or service names and various billing structures. In U.S. market, the normal format of this social network based telecom service is mobile companies' "Family Plan", which offer monthly service package for two and more family members. In Europe, this pricing strategy still remains the similar format, which offers the family package. Moreover, some telecom operators in Europe, like in Spain's Spice, offer more social network based service package, covering all family, friends, as well as working colleagues. Similar with U.S., and European countries, Japan and India also offer service monthly package for family. Instead of offering whole monthly package, telecom operators in China, such as China Mobile, offer the discounted price for each call between the parties with closed social tie. Subscribers in China have to select four telephone numbers to enjoy this discount price offer, when calling these four numbers. These subscribers usually select closed related people's number to receive this discount price offer. I select several social network based pricing examples in different countries with different styles. (See Table A)

Table A here (see Appendix)

The family based telecom service plan is becoming company's wining strategy in U.S.. A recent survey by Harris Interactive[2] shows that Americans love their wireless family plans. According to the market research firm's report, "Hot Issues Facing the Industry," two in five (41 percent) of U.S. adult mobile phone users say they currently have a wireless family plan, and of those, 97 percent say they have never had a family member leave their plan.

The social network based service in telecommunication is different from other social network based services such as family TV package, family membership, and other service packages offered to family or small groups. The consumer benefits are not only from the discounted or premium service, but also from the communication. The member in the telecom service will use the telecom service to communication with each other. The cheaper the price, the more communication will be created and the more value of consumer will be produced. The closer the relation between the communicator, the more demand for the telecom service will generate. Therefore, the lower price between the communicator with close tie or relationship will produce large consumer value and surplus. The social network based telecom service is going beyond the simple economy of scale of other types of social network based goods or service packages.

[2] Harris Interactive, (March 30, 2006), "Family Wireless Plans Prove Popular with Two in Five U.S. Adult Cell Phone Users Participating", [also available at http://www.harrisinteractive.com/news/allnewsbydate.asp?NewsID=1034]

Previous scholars studied the effects of network externalities on the firms' pricing policy in telecommunication industry (Mtichell and Vogelsang, 1991) and the study shows attracting more users to participate in the network in order to internalize network externalities. Willig (1979) analyzed that efficient pricing with network externalities is dealing with lower price for one-part pricing and a lower fixed fee for two-part tariffs (Littlechild, 1975), compared with non network effects market. Oren, Smith, and Wilson (1982) discussed that the firms have incentive to subsidize some low-demand consumers by reducing fixed fee.

Few studies have been done to connect social tie effect with network effects in pricing strategy for telecom operators, as network effects are different based on different social tie relationship. Consumers receive higher benefits when a strong-tie-related consumer joins the network than a weak-tie-related consumer. Telecom operators are able to differentiate these consumers and market higher profits through offering different tariff package to various types of consumers. Additionally, consumers could also benefit from various tariff packages that best serve their needs. Therefore, we need put the price discrimination analysis into our study.

A plenty of price discrimination studies have been done, but few of them categorize consumer with social tie relationship and analyze the firm's strategy targeting different social tie network. The evidences from telecom industry remind us that both network effects and social tie relationship play crucial roles in firm's competition strategy. Tirole (1988) and Wilson (1993) reviewed nonlinear pricing strategies. Oi (1971) studied a profit-maximizing theme park setting prices for the admission to the park (fixed fees) and for individual rides (variable fees) in a second-degree price discrimination study. Hahn (2001) discussed nonlinear pricing for telecom communication with network externality. Mitchell (1978) did a second-degree price discrimination study on local telephone service market. Shi (2003) did a similar study on social network based price discrimination study, but he overlooked an important factor, the network size of each people's various type of relationship that effect the consumer's utility and demand function. Other scholars and research did not address the connection between social network and price discrimination strategy. Shi (2003) did research on two part tariff pricing strategy include fixed fee and variable fee per minutes. Fixed line telephone market might be the case as Shi discussed. However, in this study, we more focus on mobile telecom industry and its family plan package. In this scenario, most telecom mobile operators offer one part tariff included monthly package based on minutes of usage and minutes based charge. In either case, we can consider the pricing structure as minutes based price, as monthly package is still based on minutes of usage per month. Therefore, we only consider one-part tariff pricing strategy in our model, which matches better to the market scenario.

This study has several findings that contribute to several areas. Consumers with more strong ties in their networks receive social tie-based price discount and therefore a firm may offer such a pricing policy to extract a larger surplus from consumers with more weak ties and communication with weak tie connections. Besides, social network based price discrimination strategy has been rarely studied in the previous work. In this paper, we also analyze both network size and network effect to the study as it is the crucial factor to value the consumer's utility in the certain networks. Previous studies either overlook the network effect or overlook the social network's effect on price discrimination strategy.

I begin with literature review of network effects and the relationship between social network and telecommunication in section 2. I will consider the social network into the network effects. Following the theory of "virtual network is social network" and the different social tie in the personal communication network's impact on the consumer's value of communication is different level of social network; I model the personal social network into the network effects. I use the analysis of social network and network effects to form the function of consumer utility of telecommunication services in section 3. Then I computerize the consumer demand and surplus from the utility function. In section 4, I solve the telecom operators' profit maximization problem with consideration of different utility and demand function based on consumer's different type of networks, basically the strong tie network such as family, co-worker, friends, and so on, as well as weak tie network such as university schoolmate, people randomly know, and so on. Then I will analyze the telecom operator's strategy of offering consumer different service price based on their level of social network, namely, strong tie or weak tie, and consumer surplus as the result.

2. LITERATURE REVIEW

2.1 Network Effects

Network externalities, also called network effects today, are from a basic idea that as the number of users in the network increase, the value of the network to other users also changes. When consuming a network good and service, consumers' value is not only from quality and quantity of certain products, but also from the size of a product's installed base of users (or *network*). This concept has existed in telecommunication literature for decades and in economics literature for over a century. The earliest studies about network effects include Harvey Leibenstein (1950)'s definition of "bandwagon" effects, which refer to individuals demand more a product when a larger number of other individuals demand the product. Economists define these types of products as *network goods*, and the positive relationship between the perceived value of a product and its network size is attributed to *positive consumption network externalities* (Farrell and Saloner, 1985; Katz and Shapiro, 1985). The modern network literature defined network effect as a change in the benefit, or surplus, that an agent derives from a good when the number of other agents consuming the same kind of good changes.

Farrell and Saloner (1986)'s models on network goods suggest that there is greater tendency towards monopoly and the strength of the network externalities created as a by-product of an existing installed based may lead to a bandwagon effect. Katz and Shapiro (1986) suggest that the total benefit derived from a network product or service depends in part on the number of consumers who adopt compatible products in the future. Thus, *consumers' expectations* may determine the outcome of competition in the network market.

There are also empirical researches on network effects, which focused on the context of the market for spreadsheet software (Gandal, 1994; Brynjolfsson and Kemerer, 1996). Gandal (1994) illustrated that compatibility with a dominant standard generates measurable value among spreadsheet products by showing his hedonic pricing models. In his later study (Gandal, 1995), Gandal examined standards compatibility across product categories with cases from database management systems and spreadsheets. Brynjolfsson and Kemerer (1996) extended the hedonic model for network effects, using market share as a proxy for the extent of network installed base. This work further supported the network effects hypothesis by showing that firms with a larger market share (or network) exhibited standards and quality adjusted price premiums over competitors with smaller market shares.

There are two types of network effects, direct and indirect, a distinction due to Katz and Shapiro (1994). Direct network effects have been defined as those generated through a direct effect of the number of purchasers on the value derived from a product such as fax machines. Indirect network effects are market mediated effects such as cases where complementary goods like toner cartridges, are more readily available or lower in price as the number of users of a good, laser printers, increases. Note that direct effects can exist in either literal or virtual network. For example, the number of MSN messenger users affects the value of the service and is a direct impact on a literal network. The number of PowerPoint users, on the other hand, is a direct impact, for pile transfer, at least, on a virtual network.

Although there are a lot of studies on network effects on user's value in telecommunication networks, few researches have been done on measurement of users' value in telecommunication network affected by personal social network. In the following section, I will put personal social network into the consideration of network effects and users' value in telecommunication network.

2.2 Social Network and Network Effects

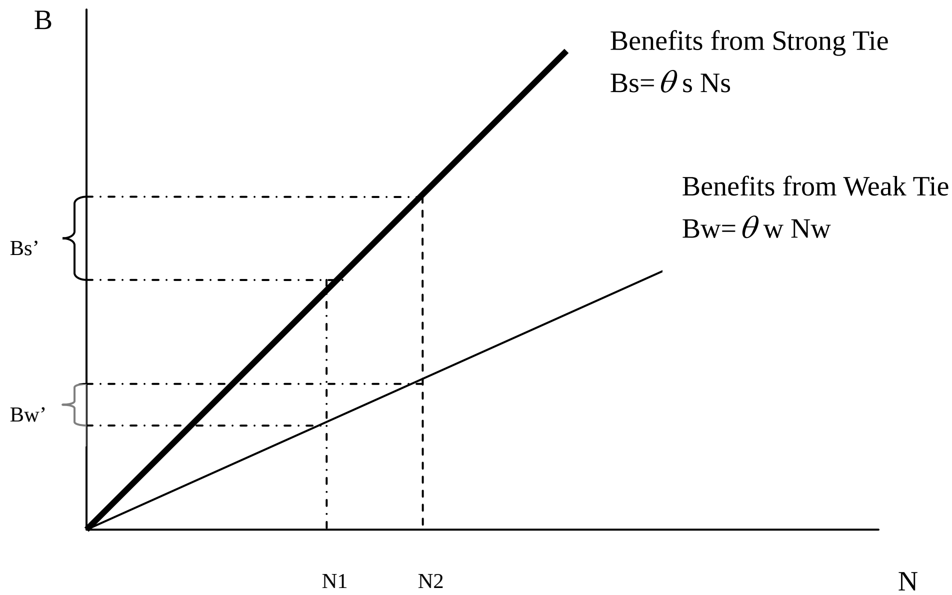
Research on the structure of social networks has found heterogeneous communication patterns both within personal networks and between personal networks, For example, in an experimental study on electronics communications among 50 social network researchers (Freeman, 1986), individuals sent a large number of messages to receivers with whom they had stronger acquaintanceship. Moreover, total number of messages that each individual sent during the period of experiment varied significantly. Dimmick et al. (1996) found that frequency of phone calls between randomly selected pairs of subjects increased with strength of their ties, which were described by proximate and affective relations. In these scholars' experimental works, the demand for communication between the people with pervious stronger relationship is more than the people with weak relationships.

Communication network is significantly shaped by personal social networks such as family, working relations, friends, and so on. Wellman (2001) points out that computer networks are inherently personal social networks. Although computer, internet, and telephone increase the personal reach to one another, the large percent of communication through these technologies are still staying in personal social networks. Studies also show that previous face-to-face relations, and work relations could increase the communication through the communication technologies such as telephone and emails. (Cummings, Butler, and Kraut, 2002)

Freeman's study (1986) shows that an individual tends to communication more with someone else with strong tie, when people do not pay for their communication, which generates more benefits from communication with strong tie relationship. From the social network studies, personal social network impact on the consumers' demands and value of communication through communication networks. We have discussed the network effects on telecommunication networks in the previous section. Consumers value more of network if more number of people in the telecommunication network. However, when considering social network's impacts on the consumers' demand and value of communication, consumers' value of network is not only based on network size, but also based on the communication counterpart.

When we shift our focus from consumption of network to consumption of each communication through the network, the personal social network's impact on consumer's value and demand becomes very important. For example, one mobile service user evaluate his or her certain call with someone else is really impacted on the relationship with this personal he or she talks with through mobile network. This user's utility would be high if talking with closed friends or family members, while his or her utility would be minimized when he or she receives a call from a sales agent who has weak tie with him or her. Moreover, the consumer's demand for the communication with someone with strong tie with this consumer is higher than the demand for communication with weak-tie person (figure 1).

Figure 1: Benefits from Different Social Tie in Network Effects



In the figure 1, let us define N as the number of subscribers in the network. Let us assume there are two types of network: strong tie network and weak tie network. The B denotes the benefits and value of consumer to the network. The n number of users in the consumer's weak tie network will generate less utility to this consumer than the n number of users in the consumer's strong tie network, namely,

$\frac{\partial B}{\partial Ns} > \frac{\partial B}{\partial Nw}$, θs > θw, where Ns refers to the number of users in user's strong tie network, and Nw refers to the number of users in user's weak tie network. θt denotes the social tie effect on the relationship between network size and the benefits that consumer receives. The marginal utility of consumer in his or her strong tie network is bigger than in weak tie network.

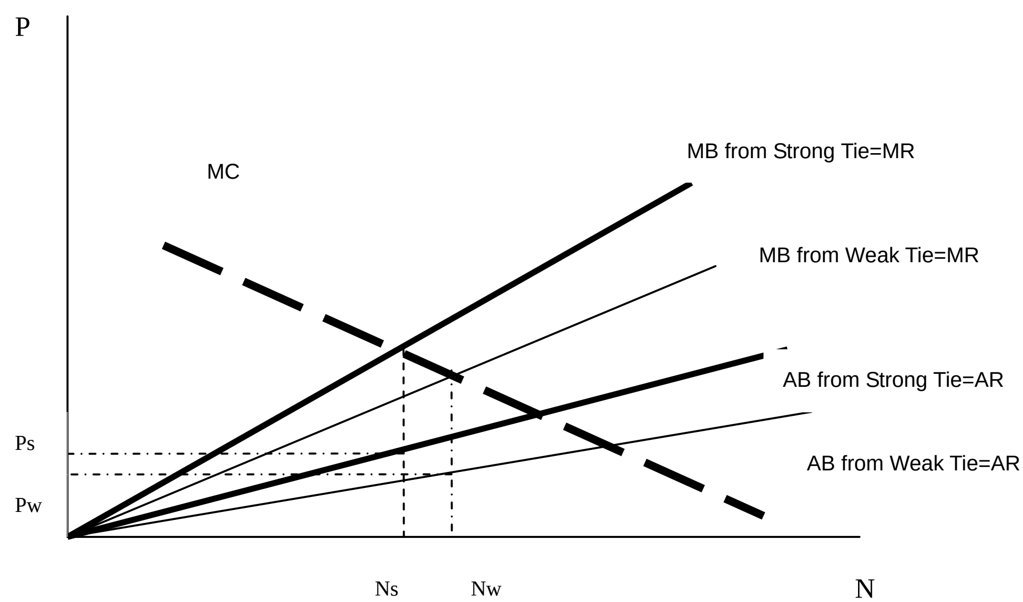
The network affects theory needs to look at the effects from social tie. The standard network effect model shows the private benefit of each network participant increases as the number of participants increases (Church and Gandal, 1993, Katz and Shapiro 1986).

With considering the social tie factors into the network effect model, I reproduce the traditional model with a decreasing marginal cost as marginal cost actually decreases in telecommunication industry with more users join the network (see figure 2).

In figure 2, the horizontal axis denotes the number of participants in some network. It can be the number of telephone users. Assume that participation in the network consists of buying one unit of some basic element of the network such as telephone service. From analysis of social tie and consumer's utility from communication, we know that marginal benefits from strong tie communication are bigger than marginal benefits of weak tie communication. We shows in the

Figure 2 that the line of MB for strong tie is above the line of MB for weak tie communication. Figure 2 also shows that MB lies above AB since an additional member raises the benefits for all network participants. The height of AB in both type s of tie networks represents each participant's willingness to pay for one unit of network communication; willingness to pay that rises as the number of network participants increases. MB, the marginal network benefit, represents the change in total benefits to network members when an additional member joins.

Figure 2:



Compared marginal cost from strong tie communication and weak tie communication, we found that firms intend to charge more for strong tie communication than weak tie communication. The application from the tradition model actually overlooks the differences in the consumer group. The traditional model only describes the network size and its relationship with the price and overlooks the each consumer's demand for communicating with users in certain types of social tie networks. While the market price for different types of social tie network is not only based on network size in each types of social tie network, but also based on the consumer's type, utility, and demand. In the following section, I will develop the demand function and utility function for each individual user in the certain type of social tie network and further model the telecom operator's price strategy targeting various types of consumers, which is categorized by whether strong tie is more than weak tie network in personal telecommunication networks.

3. MODEL AND ANALYSIS

In this section, I consider the social network in the network effects and model market demand, consumer's value for communication service within strong-tied network and weak-tied network.

Based on the consumers' demand and utility model, I will analyze telecom operators' pricing strategies in monopoly market (profit maximization) based on the consumers' different demand and value function of different types of communication networks (strong-tie and weak-tie) and operator's pricing targeting different types of consumers (consumer with many strong-tie in personal networks and consumer with few strong-tie in personal networks).

3.1 Telecommunication Market

We discussed the consumer value and demand for communication affected by social network based network effects in the section 2. I apply this idea to the telecommunication market. We assume that a market that consists of N consumers who demand for telecommunication services. The strength of social tie is presented by index variable t. Consistent with literature on social network analysis, we assume three levels of tie strength: strong ties (t=s), weak ties (t=w), and no tie (t=0) (Granovetter, 1973; Wasserman and Faust, 1994; Goldenberg et al., 2001). A consumer's utility from communications with another individual in any types of tie should be positive, but stronger the tie is, the more utility consumer will obtain duration the same spell of time normally.

In this paper, we denote the type of consumers' network by T, where $T \in \{s, w\}$. Each consumer is assumed to have two types of networks. Market consists of N number of consumers, who has n(s) number of strong ties and n (w) number of weak ties. The market then consists of two types of consumers' networks, strong-tie networks and weak-tie networks and two types of consumers: type 1 consumers with more strong-tie than weak tie in the personal networks, and type 2 consumers with more weak- tie than strong tie in the personal networks.

3.2 Consumer Value and Demand for Communication Services

3.2.1 Social Tie

The consumer's utility from each communication directly comes from the quantity of communication. When we consider the social tie into to the consumer utility function, the utility from each unit time of communication with strong tie person is larger than with weak tie person. For a pair of consumers with a tie of strength t, let $U_t(q_t)$ denote each consumer's utility from their communication of amount *q* with another consumer in the network. Let *q* denotes the quantity of communication of each consumer with another person in the network. We consider the quantity of communication q_t as the quantity of each consumer communicating with another consumer in type *t* network ($t \in \{s,w\}$, in this paper, two types of network tie will be considered: strong social tie and weak social tie).

$$U_t(q_t) = \alpha_t q_t - \frac{\beta_t}{2} q_t^{\,2} \qquad (1)$$

where $\alpha_t > 0$, $\beta_t > 0$, and t ∈ {s,w}., t refers to the strong relationships or weak relationships in certain consumer's communication network. The value of α_t and β_t depend of the strength of tie t.

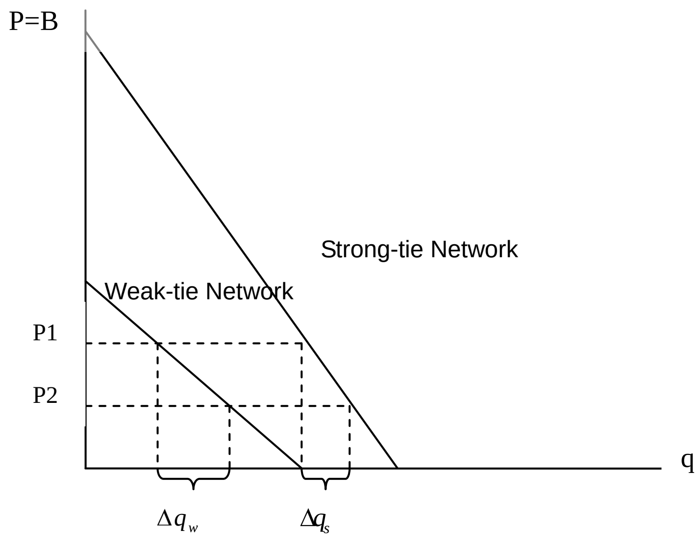
In this paper, we only consider strong tie and weak tie as two types of social networks. People tend to transmit more important information at the beginning of their conversations and their value of communication will diminish as communication lasting. As the marginal utility of communication decreases when quantity of consumption of communication in certain type of network increase, we set the nonlinear utility function in equation (1) is a concave function. The marginal utility of consumer in his or her strong tie network is bigger than in weak tie network.

The value of α_t and β_t depend of the strength of tie t. The marginal utility of communication in strong tie network is bigger than the marginal utility of communication in weak tie network, as we discussed in the end of section 2. We assume that consumer A get calls from two persons from his or her strong tie network and weak tie network, respectively. The marginal utility will first increase faster with the additional unit of time communication with strong tie person than with weak tie person, and then decrease slower than talking with weak tie person, which is

$$\frac{\partial U_s(q_s)}{\partial q_s} > \frac{\partial U_w(q_w)}{\partial q_w}; \quad \alpha_s - \beta_s q_s > \alpha_w - \beta_w q_w \qquad (2)$$

In this inequality function, we only analyze each consumer's communication and its utility that is function of quantity of communication, instead of putting consumer in a network effect scenario. Assuming communication between weak tie reaches maximum utility earlier than between strong tie, i.e. $\frac{\alpha_s}{\beta_s} > \frac{\alpha_w}{\beta_w}$, let us define $\alpha_S \geq \alpha_W, \beta_S \leq \beta_W$, as shown in figure 3, the quantity communication within strong-tie network is less sensitive to the price (price equals benefits, which denote the amount consumer is willing to pay) without considering the income effect or the cost to consumer.

Figure 3



3.2.2 Network Size

Now we begin to consider another effect on consumer's utility when consumer being with certain network service provider. As we discussed in section 2, a consumer's utility from staying in certain communication network such as T-Mobile or Verizon's mobile network, also depends on the number of users n_t in the network associated with certain type of social tie t. The utility function (1) does not include the effect of network, only indicating consumer's utility from communicating with other users in the network. With consideration of network size effect, I propose a utility function combining the effects from both quantity of communication and the network effects as follows:

$$U_t(q_t, n_t) = n_t(\alpha_t q_t - \frac{\beta_t}{2} q_t^{\,2}) \qquad (3)$$

where n_t is the total number of connections in certain consumers' various type of relationship networks; α_t >0, $\beta_t > 0$, and t $\in$ {s,w}., t refers to the strong relationships or weak relationships in certain consumer's communication network. The value of α_t and β_t depend of the strength of tie t.

The utility function (3) shows that the consumer's utility increases as the growth of users in certain type of network. As we focus the social network and network effect on telecom pricing, we now neglect the income affect. But the marginal utility should be non negative without price effect on the consumer's utility as additional unit of communication does not have negative effect on consumer's utility, i.e. $\frac{\partial U_t(q_t,n_t)}{\partial q_t} = n_t(\alpha_t - \beta_t q_t) \geq 0$; $q_t \leq \frac{\alpha_t}{\beta_t}$

We define that additional user joining the strong tie network of certain consumer will add more utility to consumer than additional user joining weak tie network.

$$\frac{\partial U_s(n_{s,} q_s)}{\partial n_s} > \frac{\partial U_w(n_{s,} q_w)}{\partial n_w};$$

Each consumer has two types of networks, and the total utility of each consumer by subscribing communication service turns to be:

$U_t(q_t, n_t) = n_s(\alpha_s q_s - \frac{\beta_s}{2} q_s{}^2) + n_w(\alpha_w q_w - \frac{\beta_w}{2} q_w{}^2)$, where n_S denotes the number of users in each consumer's strong-tie network, and n_W denotes the number of users in each consumer's weak-tie network; q_S and q_W denote the quantity of communication, namely the time of communication, in the strong-tie network and weak-tie network, respectively. p_S and p_W denote the price of communication in strong-tie network and weak-tie network, respectively.

Then in order to get optimal quantity of communication within certain type t of personal communication network, where t ∈ {s,w}, we solve following consumer utility maximizing problem with respect to q.

$$\begin{aligned} &\mathrm{q_t}(p_t, n_t) = \arg\max\{U_t(q) - n_t p_t q\}; \\ &\mathrm{q_t}(p_t, n_t) = \arg\max\{[U_s(q) - n_s p_s q_s] + [U_w(q) - n_w p_w q_w]\} \qquad (4) \\ &= \arg\max\{n_s(\alpha_s q_s - \frac{\beta_s}{2} q_s{}^2) - n_s p_s q_s + n_w(\alpha_w q_w - \frac{\beta_w}{2} q_w{}^2) - n_w p_w q_w\} \end{aligned}$$

When the optimal quantity of communication is positive, we could get an optimal quantity of communication through solving first-order condition of equation (4), and then we get following inverse demand function for communication service of a consumer in his or her type t network.

$$\begin{aligned} &\frac{\partial(U_t - p_t q_t)}{\partial q_t} = n_t(\alpha_t - \beta_t q_t(p_t) - p_t) = 0 \\ &q_t(p_t) = \frac{\alpha_t - p_t}{\beta_t} \end{aligned} \qquad (5)$$

where t ∈ {s,w}, α_t, β_t depends on the level of social tie

We get the optimal total demand of each consumer's communication with another consumer in the network. We can find that the quantity is determined by price and social tie, but not by the network size. Considering the number of consumers in the network, the total quantity of demand of each consumer becomes (assuming each consumer communicates with every consumer in same type of network for equal optimal quantity of time):

$$D(p_s, p_w) = n_s \frac{\alpha_s - p_s}{\beta_s} + n_w \frac{\alpha_w - p_w}{\beta_w}, \quad (6)$$

Where n_s, n_w denote the number of users in each consumer's strong-tie network and weak-tie network respectively, and $n_s + n_w$ =N, N denotes the total number of subscribers in the consumer's network.

Now we further detail the effects of strong tie and weak tie in equation (5) and we get function (7).

$$q_s(p_s) = \frac{\alpha_s - p_s}{\beta_s}, q_w(p_w) = \frac{\alpha_w - p_w}{\beta_w} \quad (7)$$

As we discussed about the social network's effects on communication, an individual tends to communication more with someone else with strong tie, when people do not pay for their communication (Freeman, 1986). Let us define that demand for communication in strong tie network is larger than the demand in a weak tie network for each consumer That is to say, $q_s > q_w$. Then from equation (7), we can find that: $\frac{\alpha_s - p_s}{\beta_s} > \frac{\alpha_w - p_w}{\beta_w}$. The order of the function depends on the number of consumer in the strong tie or weak tie network.

Through derivation of utility function with respect to quantity, we can find that consumers with weak tie have lower evaluation for the extra amount of communications than the consumers with strong tie. Hence, the marginal utility of communication within the network with strong tie between consumers is always bigger than weak tie between consumers, and the demand for communications within strong tie network is larger than demand in weak tie network. In the section 2, we assume that $\alpha_s > \alpha_w$, and $\beta_s < \beta_w$, which refer to that marginal value of communication declines faster within a weak tie network than within a strong tie network. The difference between communications within strong tie network and weak tie network increases, when price goes up.

In this case, the demand for communications in strong tie network is larger than demand in weak tie network, namely, $q_s > q_w$ when price is fixed. And the consumption within weak tie network is more sensitive to the price compared with the consumption within the strong tie network, which is $\Delta q_w > \Delta q_s$

Now let us consider a total consumer's demand for telecommunications, which includes a consumer's demand for communication with strong ties and with weak ties. Let us use D denote the total demand of a consumer; the total demand function is as follows:

$$D(p_{s,} p_{w,} n_{s,} n_w) = n_s q_s(p_s) + n_w q_w(p_w)$$
$$= n_s \frac{\alpha_s - p_s}{\beta_s} + n_w \frac{\alpha_w - p_w}{\beta_w} , \qquad (8)$$

where $n_s + n_w$=N, N denotes the total number of subscribers that has connection with certain consumer in the network.

Essentially, consumers' surpluses from communication should be non-negative and let us define V(P) as consumers' surplus from communication of quantity q. we substitute the equation (5) into the utility function to get consumer surpluses:

$$V_t(P_t) = \frac{n_t}{2\beta_t}(\alpha_t - p_t)^2 \quad \text{t=s, w} \qquad (9)$$

Let us go back to the assumption of two types of consumers, which are Type 1 consumers with more strong ties than weak ties in personal networks, and type 2 consumers with more weak ties than strong ties in personal networks. We could consider these two types of consumers as consumer with closed tie with family members and friends, or a consumers with loose tie with family members and don't have a lots of friends. Telecom operators are not sure if this consumer has a lot of strong tie or few strong ties in his or her personal social network. Telecom operators face the problem to maximize their profits through offering proper pricing for different types of consumers.

4 FIRM'S SOCIAL NETWORK BASED PRICING POLICY

In this section, I analyze the telecom operator's pricing strategy. When telecommunication operator already built up a communication network in certain region, telecommunication operator would face a maintenance fee when another consumer subscribes the service. Then the telecommunication operator provides communication services at a decreasing marginal cost c.

A two-part tariff is feasible when the telecommunication operator faces two types of consumers with different type of networks on the basis of the level of their personal social network. The telecommunication operator will adopt a non-linear pricing scheme consisting of a set of price (p)/social-tie type (T) bundles. If, for example, there are n level of personal social tie, then operator will publish a price/social tie level schedule of the form {(p1,T1),(p2.T2)…, (pi,Ti)..., (pn, Tn)}, which offers Ti level of personal social network with charge of pi.

Telecom operators aim to offer proper price for the right type of consumer through screening. Telecom operators define there are possibility of ρ_1 that the consumer is type one, which refers to consumers with more strong ties than weak ties in personal networks, and possibility of ρ_2 that consumer is type two, which refers to consumers with more weak ties than strong ties in personal networks.

Considering the inequality of strong tie connections and weak tie connections in different consumers, we assume there are two types of consumers for the simplicity:

Type 1 consumers: more strong-tie than weak tie in the personal networks, denoting $n_1^s > n_1^w$;

Type 2 consumers: more weak- tie than strong tie in the personal networks, denoting $n_2^w > n_2^s$;

Where, w and s refer to relationship type, 1 and 2 refer to the type of consumers

Then we modify the total demand of consumer to become:

Type 1 consumer:

$$D_1(p_{s,}p_{w,}n_{s,}n_w) = n^s{}_1 q^s{}_1(p^s{}_1) + n^w{}_1 q^w{}_1(p^w{}_1) = n^s{}_1 \frac{\alpha_s - p^s{}_1}{\beta_s} + n^w{}_1 \frac{\alpha_w - p^w{}_1}{\beta_w},$$

where $n_1^s > n_1^w \quad n_1^s + n_1^w = N$

Type 2 consumer:

$$D_2(p_{s,}p_{w,}n_{s,}n_w) = n^s{}_2 q^s{}_2(p^s{}_2) + n^w{}_2 q^w{}_2(p^w{}_2) = n^s{}_2 \frac{\alpha_s - p^s{}_2}{\beta_s} + n^w{}_2 \frac{\alpha_w - p^w{}_2}{\beta_w},$$

where $n_2^s < n_2^w \quad n_2^s + n_2^w = N$

Now we consider telecom operators to offer different price to strong-tie network and weak-tie network of certain type of consumer, i.e. offering p_1^{w} to type 1 consumers' weak-tie network, and offering p_1^{s} to type 1 consumers' strong-tie network; offering p_2^{s} to type 2 consumers' strong-tie network, and offering p_2^{w} to type 2 consumers' weak-tie network. Now we consider maximization profit problem of telecom operators. Telecom operators have to consider consumer's utility and demand function, when making price policy, in order to maximize the profits.

$$\begin{aligned}\pi &= \rho_1[n_1^s q_1^s(p_1^s)(p_1^s - c) + n_1^w q_1^w(p_1^w)(p_1^w - c)] \\ &+ \rho_2[n_2^s q_2^s(p_2^s)(p_2^s - c) + n_2^w q_2^w(p_2^w)(p_2^w - c)]\end{aligned} \quad (10)$$

Where ρ_1 and ρ_2 denote the probability of consumer types included lots of strong tie network or few strong tie in the network, and $\rho_1 + \rho_2 = 1$, N denote the total users of telecom operators. C denotes the marginal cost of telecom operators.

We obtained demand function for each type of consumer in the section 2.

$$q_1^s = n^s{}_1 \frac{\alpha_s - p_1^s}{\beta_s}; q_1^w = n^w{}_1 \frac{\alpha_w - p_1^w}{\beta_w}$$

$$q_2^s = n^s{}_2 \frac{\alpha_s - p_2^s}{\beta_s}; q_2^w = n^w{}_2 \frac{\alpha_w - p_2^w}{\beta_w}$$

Then we substitute q into function (10), and we have:

$$\begin{aligned}\pi &= \rho_1[n_1^s \frac{\alpha_s - p_1^s}{\beta_s}(p_1^s - c) + n_1^w \frac{\alpha_w - p_1^w}{\beta_w}(p_1^w - c)] \\ &+ \rho_2[n_2^s \frac{\alpha_s - p_2^s}{\beta_s}(p_2^s - c) + n_2^w \frac{\alpha_s - p_2^w}{\beta_s}(p_2^w - c)]\end{aligned} \quad (11)$$

Telecom operators' profit function faces individual rationality (IR) conditions (12) and incentive compatibility (IC) conditions (13).

IR: (1) $0 \le \frac{n_1^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_1^w)^2$

(2) $0 \le \frac{n_2^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_2^w)^2$ (12)

IC: (1) $\frac{n_1^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_1^w)^2 \ge \frac{n_1^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_2^w)^2$

(2) $\frac{n_2^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_2^w)^2 \ge \frac{n_2^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_1^w)^2$ (13)

IR (2) and IC (1) are not binding. So we have two constraints that ensure consumers with more strong social tie in their network receive zero surplus and consumers with more weak social tie in their network are indifferent between price plan (p_1^w, p_1^s), and (p_2^w, p_2^s). We obtain the following results[3]:

$$p_2^s = p_2^w = c$$

$$p_1^s = \frac{\rho_1[\alpha_s(n_1^s - n_2^s) + n_1^s c] - n_2^s\alpha_s}{\rho_1(2n_1^s - n_2^s) - n_2^s} = c + \frac{\rho_1[(n_1^s - n_2^s)(\alpha_s - c)] - n_2^s\alpha_s}{\rho_1(2n_1^s - n_2^s) - n_2^s}$$

$$p_1^w = \frac{\rho_1[\alpha_w(n_1^w - n_2^w) + n_1^w c] - n_2^w\alpha_s}{\rho_1(2n_1^w - n_2^w) - n_2^w} = c + \frac{\rho_1[(n_1^w - n_2^w)(\alpha_w - c)] - n_2^w\alpha_s}{\rho_1(2n_1^w - n_2^w) - n_2^w}$$

We assume that $(n_1^s > n_2^s), (n_2^w > n_1^w)$ because type 1 consumers have more people in both their strong-tie network and weak-tie network than type 2 consumers. Then we assume $\alpha_w < \alpha_s < c$.

We can find

$$\rho_1(2n_1^w - n_2^w) - n_2^w < 0 \text{, or } \frac{n_1^w}{n_2^w} < 1 < \frac{1+\rho_1}{2\rho_1}$$

Therefore, we know that $\frac{\rho_1[(n_1^w - n_2^w)(\alpha_w - c)] - n_2^w\alpha_s}{\rho_1(2n_1^w - n_2^w) - n_2^w} > 0$, and then p_1^w is above the marginal cost C. We also find that p_1^s is less than marginal cost C, if $\rho_1(2n_1^s - n_2^s) - n_2^s > 0$ or $\frac{n_1^s}{n_2^s} > \frac{1+\rho_1}{2\rho_1}$.

[3] See Appendix 2

Solving telecom operator's profit maximization problem, we find that optimal prices for the consumer with lots of strong ties, namely type 1 consumer are more than marginal cost for communication within the weak tie network. However, the price for type 1 consumers' communication within strong tie depends strength of ties, namely α , and number of users in each type of network, namely n_1^t and n_2^t, t=(s,w). We can not only find that network size's effects on consumer's utility and firm's pricing strategy, but also find the social tie is crucial to determine the consumer's utility and firm's pricing stragey. The probability is large that price for strong tie communication of type 1 consumer is less than marginal cost, which can explain the real market situation and could explain well the family plan and those lower price strategy for communication with friends adopted by telecom operators.

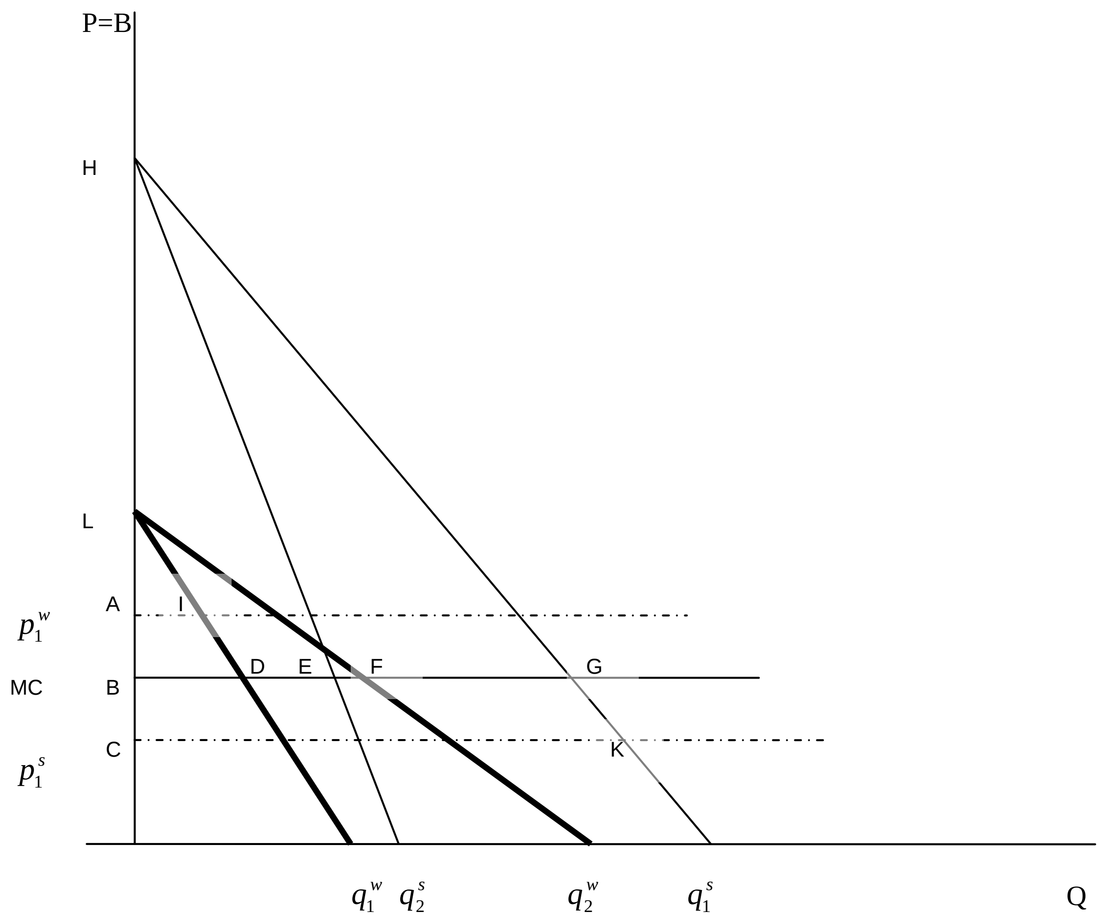
The variable price charged for types 2 consumers equal marginal cost of telecom operators. From here, we could find it is reasonable for telecom operators to charge the consumers with lots of strong ties lower price for communication within their strong tie networks and higher price for communication within their weak tie network. Our findings are consistent with standard second-degree price discriminations using a menu of two-part tariffs (see Tirole, 1988; Wilson, 1993). In general, the optimal variable prices for the high-valuation segment (type 2 consumers) are equal to marginal cost. However, our finds is on the contrast to the traditional model, which shows the price for strong tie network is higher than for weak tie networks. We summarize these results in the following proposition.

Proposition 1: We define $\alpha_S \geq \alpha_W, \beta_S \leq \beta_W$, and the optimal variable fees are equal to marginal cost for the consumers with more weak ties than strong ties in their network. For the consumers with more strong ties than weak ties in their networks, the optimal variable fee is above marginal cost for communication within weak ties but below marginal cost for communication within the strong ties.

Telecom operators offer two distinctive pricing plans targeting two consumers groups. Our results suggests operators can set price at marginal cost for the consumers with more weak tie in their networks than strong tie, but deviate from marginal cost fro the consumers with more strong tie in their networks. Telecom operators' deviations reduce the profits from the consumers with more strong ties in their networks, namely type 1 consumer, but extract more surpluses from those consumers with more weak ties in their network, namely type 2 consumers. The firm could increase the profits from the type 2 consumers, which can dominate the loss of profits from the type 1 consumers.

We further analyze the consumer surplus and firm's profits in figure 3

Figure 3: Aggregate Demand Function for Strong Tie and Weak Tie



In Figure 3, line Hq_1^s refers to aggregate demand ($n_1^s q_1^s$) of type 1 consumer for communication within strong tie network. Line Hq_2^s refers to the aggregate demand ($n_2^s q_2^s$) of type 2 consumer for communication within strong tie network. Line Lq_1^w refers to aggregate demand ($n_1^w q_1^w$) of type 1 consumer for communication within weak tie network. Line Lq_2^w refers to aggregate demand ($n_1^w q_1^w$) of type 2 consumer for communication within weak tie network.

We assume that telecom operators charge both types of consumer marginal cost initially as shown in figure 3 the line of BG. Type 1 consumer surplus from communications is BGH for strong tie and BDL for weak tie. Type 2 consumer surplus from communications is BEH for strong tie and BFL for weak tie. We found that the surplus of type 1 consumer is bigger than type 2 consumer, i.e. V1>V2, or BGH+BDL>BEH+BFL.

We move the price for weak tie communication from marginal cost up to line A, i.e. p_1^w. Since a type 1 consumer has less weak ties, increasing fee for the weak ties to decrease communications reduces a type 1 consumer's service valuation less than type 2 consumers. Therefore, increase p_1^w only slightly decrease type 1 consumer's surplus. We also can find that moving price for strong tie communication from marginal cost down to line B, i.e. p_1^s can increase the type 1 consumer's surplus. Therefore, if telecom operators provides p_1^s and p_1^w to the type 1 consumer, the type 1 consumer's surplus will largely increase from BGH+BDL to CKH+AIL.

5 CONCLUSIONS

In this paper, we have molded the monopoly firm's optimal social network-based discriminatory pricing strategy. Our model illustrates that the optimal pricing strategy depends on the structure of social networks and the different level of social relations, or ties. We find that the optimal price for the consumer with more strong ties is less than marginal cost to communicate with strong tie and larger than marginal cost to communicate with weak ties. Additionally, the optimal price for the consumer with less strong ties equals the marginal cost. Telecom operators could offer different prices for these two types of consumers.

Currently, most mobile operator did not differentiate the price between different types of consumers and offer same tariff package included price for strong tie network such as family plan and cheaper price for friends communication, and price for weak tie network, just normal minutes based price. Based on what we found from the model, mobile operators can adopt new strategy in addition to currently family plan to offer multiple packages targeting different consumer group. For consumer with lots of strong tie connections, operators can charge less low price for talking with friends and within consumer's strong tie network and increase the price for communication within the weak tie network. For consumer with few strong tie connections like only communicating with family members or limited other strong tie connections, operators could offer normal package based on the marginal cost. As we know the marginal cost in the telecom industry is decreasing when more joining the network, operators can actually increase its profits by attracting more consumers to its network. Different tariff packages also benefit consumer with different type of social tie networks.

This is the start point research and further research needs to analyze completive market as well as the social-network pricing strategy's impact on consumer lock-in effects.

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APPENDIX 1

Table A: Social Network Based Telecom Service Pricing

Telecom Carriers	The service Name	Normal Tariff	Family or friend tariff plan	Service scope
China Mobile	Relatives Number	RMB 0.60/min (USD 0.075/min)	RMB 0.24/min (USD 0.03/min) -supper relative number RMB 0.36/min (USD 0045/min) -Normal relative number	1 supper relative number and 4 normal relative number/ per subscriber
NTT DoCoMo	Discount on basic monthly charge		25%	A great discount for families and individuals using 2 or more DoCoMo lines. Save on both basic monthly charges and dialing charges
	Discount on communications and Videophone communications charges between family members		30%	
	Discount on communications charges to a designated landline phone		10%	
Fiji Telecom	Local	$ 0.1227/call	$ 0.0563/call	54%
	Intra-region	0.1452/min	0.1125/min	23%
	Inter-region	0.2306/min	0.1800/min	22%
	Mobile	0.5569/min	0.2475/min	56%
Verizon	Family Share 700	America's Choice 450 $39.99 Anytime Minutes: 450	$69.99 (two lines) Anytime minutes: 700	Unlimited IN Calling AND Night & Weekend Home Airtime Minute
	Family Share 1400		$89.99	
	Family Share 2100		$109.99	
Spice	Spice Quicky Team Plan		Team Members | Saving 2 | 50%	

			3	67%	
			4	75%	
			5	80%	
			6	83%	
Vodafone		Monthly fixed cost - only 59€ per month	Monthly Acess Fee of main connection with 1 additional member only 77.945€ per month Monthly Acess Fee of main connection with 2 additional members only 85.68€ per month Monthly Acess Fee of main connection with 2 additional members only 93.415€ per month		New Family tariff plans offer to Family members : up to 4 connections with one monthly acess fee and one bill the flexibilty to share the Free of charge minutes 50% additional talk time for the inta family calls
Cingular		Cingular FamilyTalk Nation 1400 Rollover Minutes &80.00	Cingular Nation 450 Rollover Minutes $ 39.99		
T-Mobile	Family Plan	T-Mobile FamilyTime 1000 Minute Plan with Unlimited Nights and Weekends $69.99	T-Mobile Get More 600 Nationwide Minute Plan with Unlimited Nights and Weekends $ 39.99		Free in-network minutes
AirTel India	AirTel India Family Pack(National) Chest 450		50 Free local calling minutes to each family member; 25 Free local SMS		
	Chest 1000		75 Free local calling minutes to each family member; 25 FREE M2M STD minutes for each member for calling within		

			the family	
CHENNAI TELEPHONES	Friends & Family Talk Plan-325		50% of the normal call charges in the respective plan	

Source: company's websites (2006)

APPENDIX 2

Maximizing firm's profit function subject to IR and IC constrains:

$$\pi = \rho_1[n_1^s \frac{\alpha_s - p_1^s}{\beta_s}(p_1^s - c) + n_1^w \frac{\alpha_w - p_1^w}{\beta_w}(p_1^w - c)]$$

$$+\rho_2[n_2^s \frac{\alpha_s - p_2^s}{\beta_s}(p_2^s - c) + n_2^w \frac{\alpha_s - p_2^w}{\beta_s}(p_2^w - c)]$$

Telecom operators' profit function faces individual rationality (IR) conditions and incentive compatibility (IC) conditions.

IR: (1) $0 \le \frac{n_1^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_1^w)^2$

(2) $0 \le \frac{n_2^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_2^w)^2$

IC: (1) $\frac{n_1^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_1^w)^2 \ge \frac{n_1^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_2^w)^2$

(2) $\frac{n_2^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_2^w)^2 \ge \frac{n_2^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_1^w)^2$

IR (2) and IC (1) are not binding. So we have two constraints that ensure consumers with more strong social tie in their network receive zero surplus and consumers with more weak social tie in their network are indifferent between price plan (p_1^w, p_1^s), and (p_2^w, p_2^s)

$$\frac{n_1^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_1^w}{2\beta_w}(\alpha_w - p_1^w)^2 = 0$$

$$\frac{n_2^s}{2\beta_s}(\alpha_s - p_2^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_2^w)^2 = \frac{n_2^s}{2\beta_s}(\alpha_s - p_1^s)^2 + \frac{n_2^w}{2\beta_w}(\alpha_w - p_1^w)^2$$

We drive the first-order conditions with respect to the various prices for type 1 and type 2 consumers.

$$\frac{\partial \pi}{\partial p_1^s} = \rho_1 [n_1^s \frac{\alpha_s - 2p_1^s + C}{\beta_s}] + n_1^s \frac{2p_1^s - 2\alpha_s}{2\beta_s} + \rho_1 [n_2^s \frac{p_1^s - \alpha_s}{\beta_s}]$$

$$\frac{\partial \pi}{\partial p_1^w} = \rho_1 [n_1^w \frac{\alpha_w - 2p_1^w + C}{\beta_w}] + n_1^w \frac{2p_1^w - 2\alpha_w}{2\beta_w} + \rho_1 [n_2^w \frac{p_1^w - \alpha_w}{\beta_w}]$$

$$\frac{\partial \pi}{\partial p_2^s} = (1-\rho_1)[n_2^s \frac{\alpha_s - 2p_2^s + C}{\beta_s}] + n_2^s \frac{2p_2^s - 2\alpha_s}{2\beta_s} + \rho_1 [n_2^s \frac{p_2^s - \alpha_s}{\beta_s}]$$

$$\frac{\partial \pi}{\partial p_2^w} = (1-\rho_1)[n_2^w \frac{\alpha_w - 2p_2^w + C}{\beta_w}] + n_2^w \frac{2p_2^w - 2\alpha_w}{2\beta_w} + \rho_1 [n_2^w \frac{p_2^w - \alpha_w}{\beta_w}]$$

Then we can obtain the following solutions:

$$p_2^s = p_2^w = c$$

$$p_1^s = \frac{\rho_1[\alpha_s(n_1^s - n_2^s) + n_1^s c] - n_2^s \alpha_s}{\rho_1(2n_1^s - n_2^s) - n_2^s} = c + \frac{\rho_1[(n_1^s - n_2^s)(\alpha_s - c)] - n_2^s \alpha_s}{\rho_1(2n_1^s - n_2^s) - n_2^s}$$

$$p_1^w = \frac{\rho_1[\alpha_w(n_1^w - n_2^w) + n_1^w c] - n_2^w \alpha_s}{\rho_1(2n_1^w - n_2^w) - n_2^w} = c + \frac{\rho_1[(n_1^w - n_2^w)(\alpha_w - c)] - n_2^w \alpha_s}{\rho_1(2n_1^w - n_2^w) - n_2^w}$$