

**Designing a Mechanism for Reallocating Spectrum  
as a Resource with Vested Right, Sunk Cost, and Externalities<sup>1</sup>**

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**I. Introduction**

This paper attempts to design and analyze an economic mechanism of spectrum trade for its efficient use. The paper first points out that, because of the vested right, sunk cost, and technological externalities accompanying the spectrum use, market mechanism, if introduced spontaneously along the line suggested, e.g., by Kwerel and Williams [2002], would not work well as it would with ordinary goods or services. The reason is that the chance is small for a potential user of spectrum to find an incumbent who would agree to yield his/her spectrum block with vested right obtained, and the sunk cost invested, in the past. Further, even if the potential user can find such an incumbent, he/she will act as a local monopolist to the potential user and likely quote an extremely high price for (i.e., hold up) the spectrum.

We note that, for efficient use of spectrum blocks, it is necessary that both the demand price and the supply price of the block be revealed truthfully in some way; otherwise, it is impossible to judge whether a particular trade is, or is not, Pareto-improving. The demand price of a spectrum block may be obtained through auction. There is no mechanism, however, for having the incumbent reveal the supply price truthfully.

To overcome this difficulty, the paper proposes an “extended market mechanism” for spectrum trade in which each user of spectrum be obliged to reveal the supply price of his/her spectrum block, that is, to declare an amount, however large, for which the user is willing to yield the right of using the spectrum. Further, if an offer is made at a price at least as high as the supply price, then the user be obliged to trade the spectrum to the party making the offer.

Second, each user pay to the government annually a “spectrum-holding fee” equal to the product of the supply price and the fee rate, the latter to be determined by the government. This is to prevent the user from claiming an unjustifiably high supply price; further, the fee may be regarded

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<sup>1</sup> Preliminary draft; please do not quote without a permission by the author.

as a “tax” on using spectrum, which is the property of the society as a whole. Thus, in this mechanism, each spectrum user would either continue to use his/her spectrum block with the annual payment or cease using it for once-and-for-all compensation.

In the following, we will formulate such a mechanism into an economic model and analyze it to find how it works under various conditions. Welfare implications will also be investigated. Further, the paper considers how to deal with a block of spectrum used as commons within the framework of this mechanism, and how to form a supply price when indirect spectrum users such as subscribers to mobile telephony or wireless Internet are involved. Finally, the paper proposes a strategy according to which the current command-and-control mechanism may be transformed gradually, in contrast to big-bang, into the proposed market mechanism.

This paper is an extension of Oniki [2004], in which a mechanism was given to reallocate spectrum through forced acquisition (i.e., taking) by the government; this paper intends to propose a mechanism for spectrum trading in general.

## II. Spectrum as an Economic Resource and Impediments to Spectrum Trade

### A. Physical properties of spectrum

We begin with considering physical properties of spectrum. First of all, spectrum is a non-reproducible natural resource. It is different from oil or mineral deposits in that it does not deplete. It is different from produced capital like machines and equipment in that it does not depreciate. Spectrum, however, is a resource of limited supply with boundaries and a size.

In order to understand spectrum, it is useful to consider its resemblance to *land* as a resource. Land is a non-reproducible, non-depletable natural resource with limited supply; in addition, a piece of land has boundaries and a size. In fact, both land and spectrum as economic resources can be grouped into a category of *space resources*, of which examples are land space, water space, air space, the space of satellite orbits, to name a few. The resemblance of spectrum to land is a consequence of the fact that the utility of land arises from using a portion of the surface of the earth physically, whereas the utility of spectrum arises from using a portion of the surface of the earth electro-magnetically<sup>2</sup>. Thus, the term “spectrum” means, in many cases, not electro-magnetic waves themselves, but a space for electro-magnetic waves to propagate through. In this paper,

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<sup>2</sup> For simplicity, this paper considers terrestrial spectrum only. That is, we deal only with the space which is a portion of the surface of the earth used for the transmission of electro-magnetic waves, not with other spaces such as airspace or the space of satellite orbits used likewise. It is straightforward, however, to extend the discussion of this paper to other spectrum spaces.

therefore, we will use “spectrum” and “spectrum space” interchangeably.

A remarkable difference between land and spectrum is that, whereas there is only one piece of land attached to an area of the surface of the earth, there can be many spectrum spaces attached to such an area. In other words, spectrum has one more dimension, the frequencies of electro-magnetic waves. We state that land space is of two dimensions and spectrum space of three dimensions, of which two for the surface of the earth and the remaining one for the frequencies.

In this paper, we use the term (*spectrum*) *band* to represent a segment of the axis measuring frequencies. For example, the UHF television band represents the frequencies ranging from 470MHz to 770MHz (width of 300MHz), and the band of television channel 20 is the frequencies ranging from 512MHz to 518MHz (width of 6MHz). See Figure 1.

We next define the term (*spectrum*) *block* to be a subset of the three-dimensional spectrum space composed of a band of frequencies and an area of land. See Figure 2, in which is shown a spectrum block composed of an area A of land and a frequency band B; such will be denoted as block (B, A). Further, Figure 3A illustrates ten spectrum blocks neighboring each other areawise and/or frequencywise, where, for simplicity, the three axes of the entire spectrum space are not shown explicitly. Note that Figure 3B shows the six areas  $A_1$  through  $A_6$ , and the two bands  $B_1$  and  $B_2$ , used for composing the ten blocks in Figure 3A.

## **B. Regulation of spectrum use**

We note that, in many countries, the utilization of radio spectrum is administered by government in two stages; the first is *allocation* of spectrum bands for specific objectives and the second is *assignment* of spectrum blocks to users.

Allocation of spectrum is done in two levels, international and national. International allocation of spectrum is conducted by the International Telecommunication Union (ITU) and by other international bodies. The national level of spectrum allocation is made by national government, which specifies one or more objectives for using a spectrum band in more detail together with technological specifications including the power of radio emissions, the allowance of interferences, and the format of modulation and coding needed for information transmission.

The second stage of managing spectrum is assignment of a spectrum block to users. The entire spectrum space is divided into a number of spectrum blocks, to each of which a single user or multiple users are assigned with or without a license. An assignment also specifies the time (of a

day, a week, etc.) in which spectrum is used together with detailed technological specifications.<sup>3</sup>

In order to introduce extended market mechanism for spectrum trade, we first divide the entire process of spectrum regulation into two parts, part A and part B. See Figure 4. In short, part B is concerned with determining who uses each spectrum block. Part A includes everything else. Thus, in part A, a government agency responsible for regulating spectrum (the government) determines how to divide the frequencies into bands and how to divide the spectrum space into blocks with technological details including “for what purpose(s)” and “how” a spectrum block is used. Part B determines “by whom” only.

The objective of this paper is to design and analyze an economic model (extended market mechanism) for part B; we will not deal with part A in this paper. We point out, however, that information generated in part B such as prices used for trading spectrum blocks should be useful for the government engaged in part A activities. See Figure 4.

### **C. Impediments to spectrum trade**

The question of efficient use of spectrum limited to part B as stated above is how to designate a user to each spectrum block so as to maximize the total outcome from the entire spectrum resource. A textbook answer to this question is that, if spectrum trade is introduced, then each block will be held by the user who can offer the highest price for it, that is, the user who can use it most efficiently. This would be true if a spectrum block were an ordinary commodity like foods or clothing for which substitutes are available in the market and for which ample competition can be expected to hold both on the demand side and the supply side of the market. Actually, however, a spectrum block, as an object for trade, is quite different from ordinary commodities; we cannot expect that mere introduction of spectrum trade can lead us to its efficient use. We will look into this issue more in detail by considering the behavior of incumbent and potential users of spectrum.

Let us first consider potential users (Y) of a spectrum block. Suppose that Y has a business plan for using the block with a net present value of the expected stream of net profits which will be obtained in the future once the plan becomes reality. This present value is the greatest amount of money that Y can offer for obtaining the right to use the block. This is called the *demand price* for

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<sup>3</sup> Until this point, we have not mentioned of spectrum commons, an important mode of using spectrum. In the ordinary setting, commons is designated as a spectrum band which is open to the public for free use; no assignment is needed for commons. In our setting, however, for a reason to be explained later, spectrum commons is realized by assigning a block, which is large areawise, to a public agent and then having the agent let the block be used freely. Note that, in this setting, the agent can be regarded as one of spectrum users.

the block.

Note that, usually, it is more advantageous to use more than one blocks of spectrum located near-by areawise and/or frequencywise jointly as depicted in Figure 3A than to use each of them separately; this is called (positive) externalities in using spectrum. As a consequence, Y may have a demand price attached to a combination of spectrum blocks. Such pricing may become complicated even with a single potential user. In case of multiple potential users, their pricing will be further complicated.<sup>4</sup>

We now consider an incumbent user (X) of a spectrum block who faces offers with combinational pricing by multiple potential users. For simplicity, we do not introduce combinational factors with X; we assume that X considers whether to sell the right to use the block, and if so, at what price.

For this, X need to calculate two values. The first is the net present value of using the block, which is the discounted sum of the stream of expected profits from the block in the future. The second is the highest in a set of values each of which is the net present value from some project to be conducted by X without using the spectrum block, plus a once-and-for-all cost needed for the transition to a project. The difference between the first and the second values is the minimum amount of money for which X will agree to yield the right of using the spectrum block to a potential user; we call this the *supply price* of the block by X.

Observe that, if X has physical and other capital invested in the past for using the block and if such investment has become sunk cost (i.e., recovery of investment is difficult), then the supply price of the block will be very high, since X would have to start everything from scratch for a business without using the block. This situation is described that X has a vested right on the block.<sup>5</sup>

Now, consider a “market” in which incumbent users of spectrum meet with potential users for trading spectrum blocks. Suppose that the market is decentralized; that is, incumbent and potential users meet randomly one after another to negotiate for possible trade. Because of positive externalities in using spectrum blocks, potential users tend to attach a high price to a favorable combination of blocks. There is no predetermined way to divide a price attached to such a

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<sup>4</sup> Combinational auction is an algorithm which can determine a winner of each block auctioned in the presence of multiple potential users with combinational pricing.

<sup>5</sup> Note that what we are dealing with here is “trade of spectrum,” not “trade of a business in which spectrum is used.” In reality, of course, the latter is seen frequently, perhaps contributing to improve the efficiency of using spectrum blocks indirectly through transferring them to better management. We have no objection to the fact that such is possible, but it is not our target here. We are seeking the possibility of trading spectrum itself separated from capital and management attached to it.

combination into each block composing it. It is possible, therefore, that a potential buyer offers a very high price to a block which is the last one to form a favorable combination.

Suppose further that the incumbent user is allowed to offer a price for his/her spectrum block not necessarily equal to the (truthful) supply price; that is, he/she may 'lie' on it. Then, we expect that the incumbent user, seeking to sell his/her block as "the last block composing a combination of blocks favorable to some potential user," will exhibit very high price, untruthful supply price. (See Figure 5AB, in which spectrum block is represented as one-dimensional entity.) In such a case, potential buyers will be cautious in getting into trade actually; they will try to avoid paying excessive prices. As a consequence, the level of trade in such a market is expected to be far lower than the level corresponding to the one of Pareto-improving, not to mention Pareto-optimal.

### III. Extended Market Mechanism

#### A. Outline of extended market mechanism

We have just found that spectrum market will not function well by merely allowing incumbent and potential users to trade. In this section, we attempt to design an *extended market mechanism* (EMM) which can facilitate spectrum trade toward a Pareto-optimal state.

Observe that EMM should at least be able to have each incumbent reveal the supply price of his/her spectrum block truthfully, since, without truthful supply prices, it is impossible to judge whether a particular trade is, or is not, Pareto-improving. Further, EMM should also be able to centralize the information about (truthful) supply prices so that each potential user can find, if any, an incumbent with whom a Pareto-improving trade may be realized. In other words, EMM should be an organized market, in which information pertinent to Pareto-improving trade is collected and distributed systematically. For this, we need a market organizer (market regulator).<sup>6</sup>

Figure 5 outlines such EMM. The supply side of EMM is the incumbent users (X) of spectrum blocks. Each incumbent is obliged to reveal the supply price ( $C$ ) of his/her block truthfully, and the spectrum holding fee is imposed to incumbents for this.

Potential users (Y) of spectrum blocks can access, if they so choose, to the supply price of each block. Further, if Y wishes to obtain the right to use a block, Y can enter into an auction for it. Y will actually acquire the spectrum if he/she wins the auction.

Finally, Government (Z) operates the market. Z specifies a spectrum-holding fee rate ( $r$ ), and

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<sup>6</sup> An example of organized (centralized) market is stock market. Note, however, that for stock market one need not worry about the possibility of untruthful price revelation, since there is no externalities between stocks.

collects spectrum fees ( $R$ ) from each incumbent. In addition,  $Z$  holds an auction for each spectrum block with one or more potential users seeking the right of using it. If a winning bid (the highest demand price) exceeds the supply price of a block,  $Z$  receives the difference (surplus) between the demand and the supply prices. Both spectrum holding fees and auction surplus should be treated as a property tax by the general government.

## **B. Revelation of supply price by incumbents**

This section gives an outline of EMM. In short, it is a system by means of which the supply price of spectrum blocks is revealed by incumbent users. This will be realized through a mechanism of insurance and compensation.<sup>7</sup>

First of all, to incumbent spectrum users, the system EMM is a mandatory insurance with compensation. Thus, each spectrum user (including government and other public users) declares a monetary amount of *compensation* to be paid to the user in the event that the spectrum block being used by the user becomes unusable. Each spectrum user pays an insurance premium (*spectrum holding fee*) to the government annually, which is equal to the amount of compensation multiplied by the *rate of spectrum holding fee* to be determined by the government. Thus,

(spectrum holding fee)

$$= (\text{spectrum holding fee rate}) * (\text{compensation amount declared}).$$

To spectrum users, the system is nothing but a casualty insurance plan, where a casualty here is the event that the spectrum block becomes unusable.

With proper incentives, of which details are discussed later, spectrum users in general tend to declare, and risk-averse users will always declare, an amount of compensation so that it is equal to the least amount of money they can accept as compensation in the event that the spectrum block becomes unusable. This means that, in effect, the amount of compensation declared by a spectrum user is in fact the *supply price* of spectrum with regard to reallocation.

Figures 7A and B illustrate the supply side of EMM. In the figures, the horizontal axis measures the size of spectrum blocks, and the vertical axis the unit supply price of spectrum, which is equal to the amount of compensation divided by the size of a block. In Figure 7A, we express each spectrum block by a rectangle in the following way. First, the width of a rectangle is equal to

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<sup>7</sup> Ikeda and Ye [2003] proposed a system of “reverse auction” for spectrum reallocation, in which the supply price of spectrum is revealed by incumbent users who attempt to “sell” the right to use spectrum to the government at a price as high as possible. Their system, however, does not have a provision of insurance and compensation.

the size of the block, and the area to the amount of compensation declared with that block. The height of the rectangle then expresses the supply price of the block with regard to reallocation. The rectangles are arranged from left to right in the increasing order of the supply prices.

By combining the top of the rectangles, we obtain the supply curve of spectrum blocks with regard to reallocation, as shown in Figure 7B. Note that, in Figures 7AB, we neglect the location of spectrum blocks on the surface of the earth and, instead, treat them as if one-dimensional entity. This is merely for simplification.

We further note that, given a *fair* insurance program in which the premium rate ( $r$ ) is equal to the probability ( $\pi$ ) of the event that spectrum trade takes place, a rational risk-averse user will choose a *complete* insurance plan in which the utility in the event of trade be equal to the utility in the event of no trade.<sup>8</sup> (To be continued.)

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<sup>8</sup> See, e.g., Mas-Colell, et al. [1995], pp.187-188.

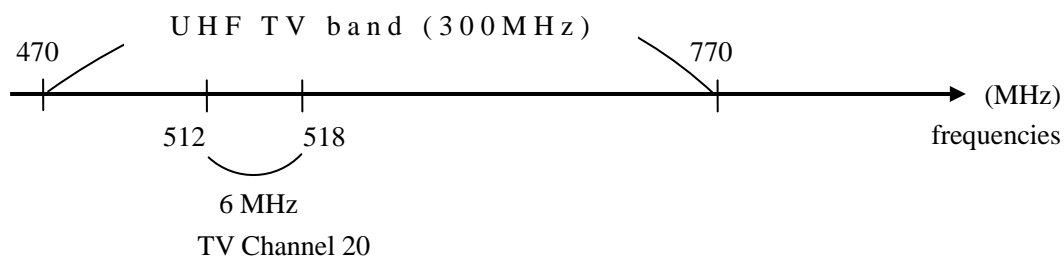
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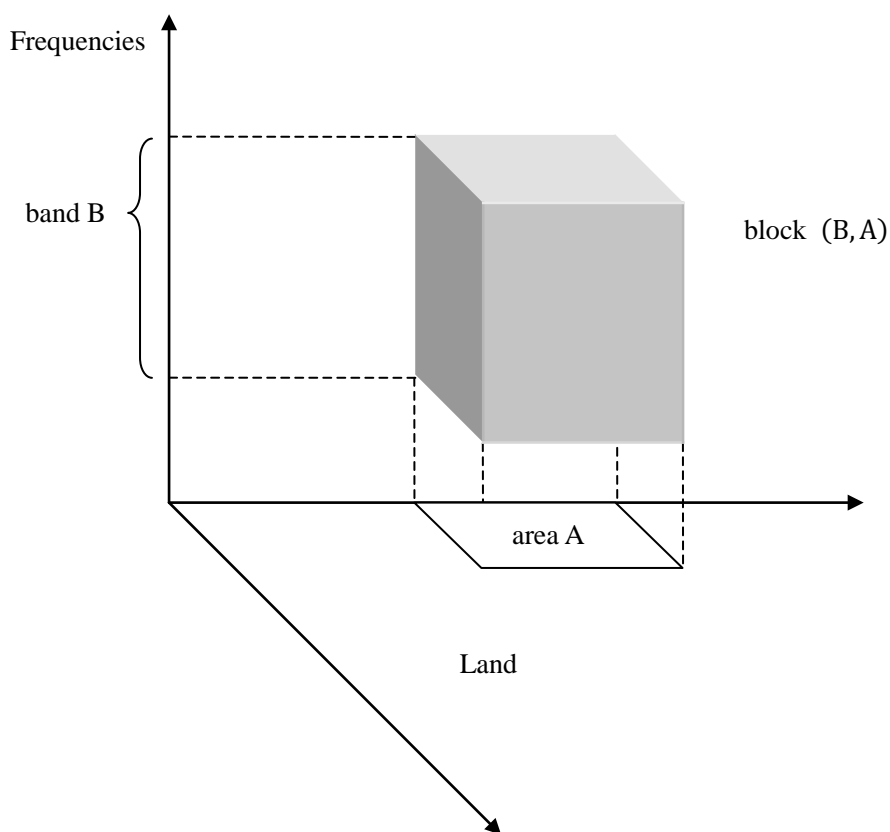
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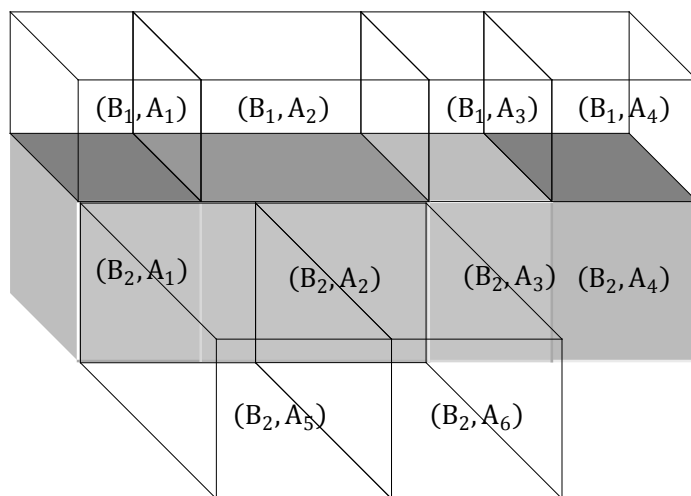
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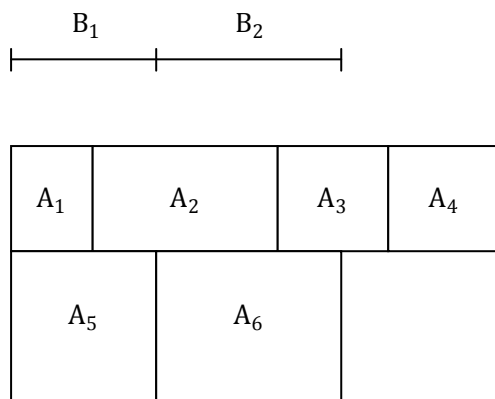
**Figure 1:** Examples of Spectrum Band in the Frequencies Axis



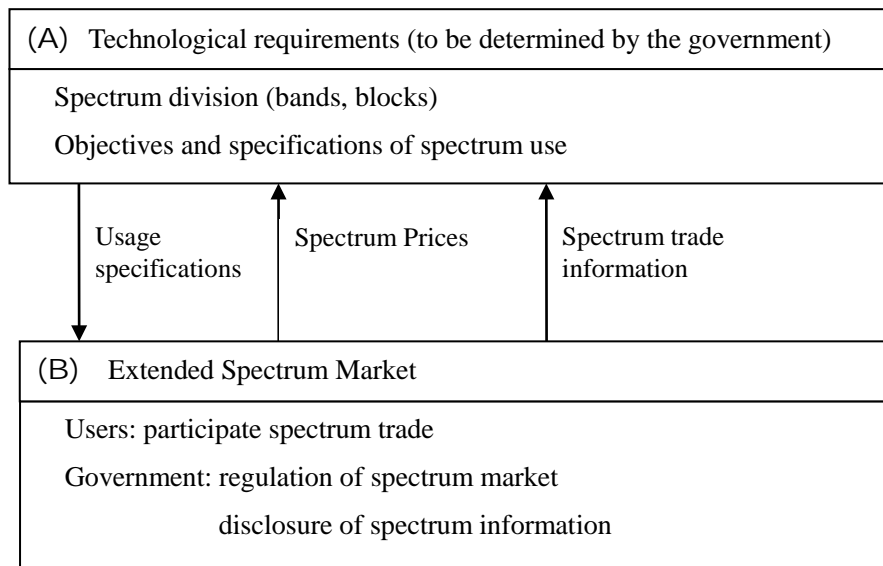
**Figure 2:** Example of Spectrum Block(B, A) in the 3-dimentional Spectrum Space



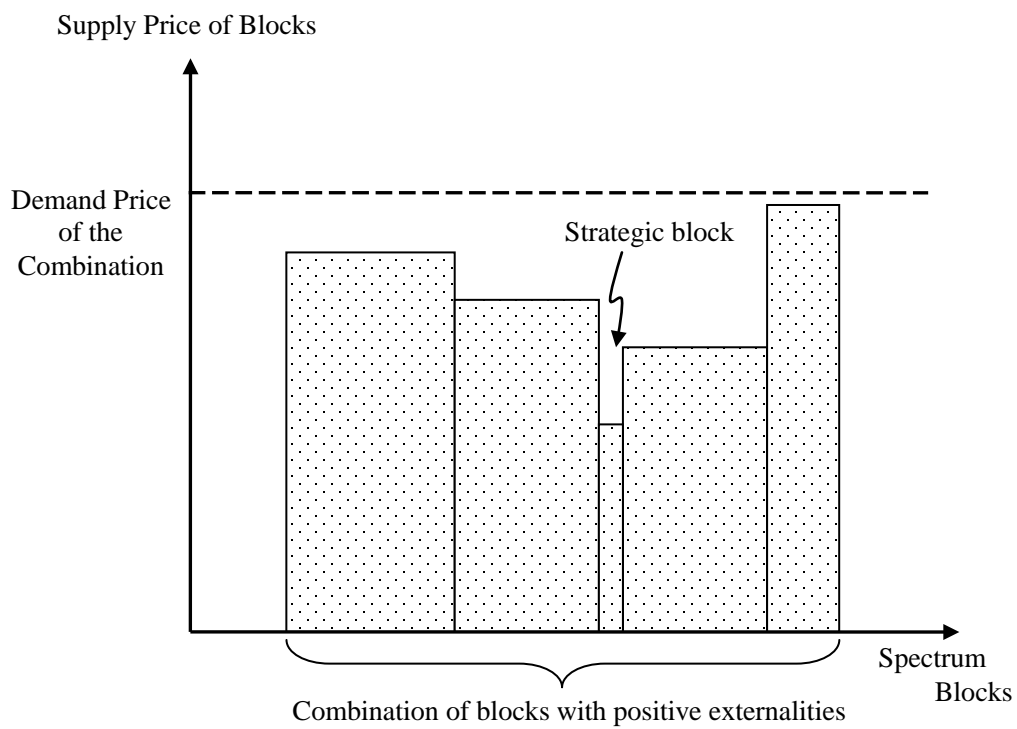
**Figure 3A:** Example of 10 Spectrum Blocks



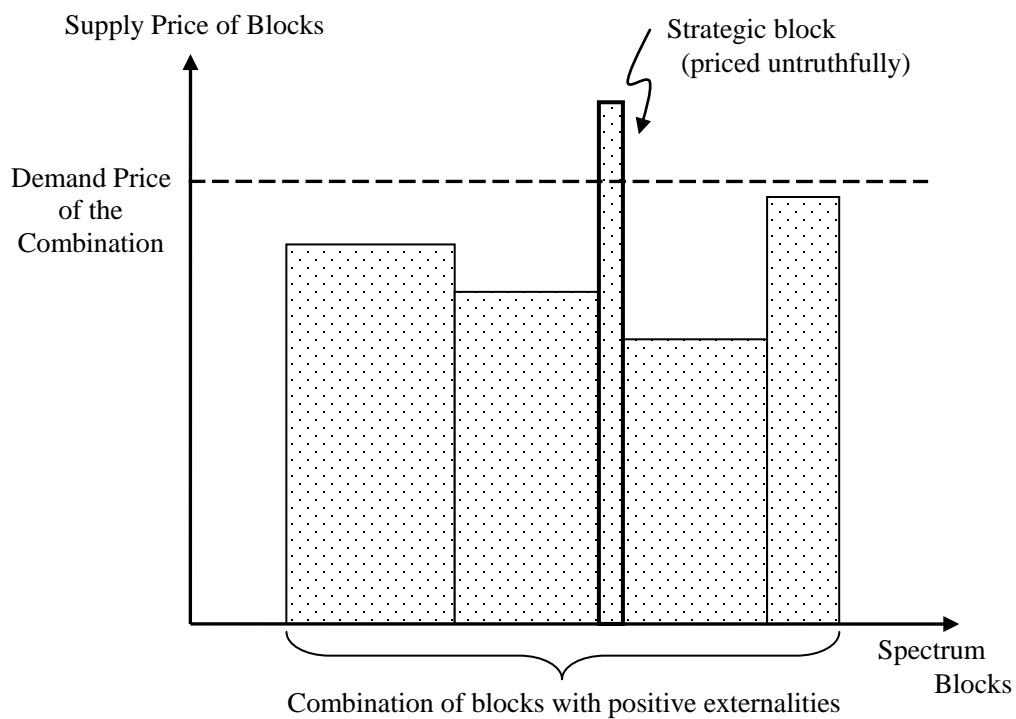
**Figure 3B:** 2 Bands and 6 Areas for the Blocks of Figure 3A



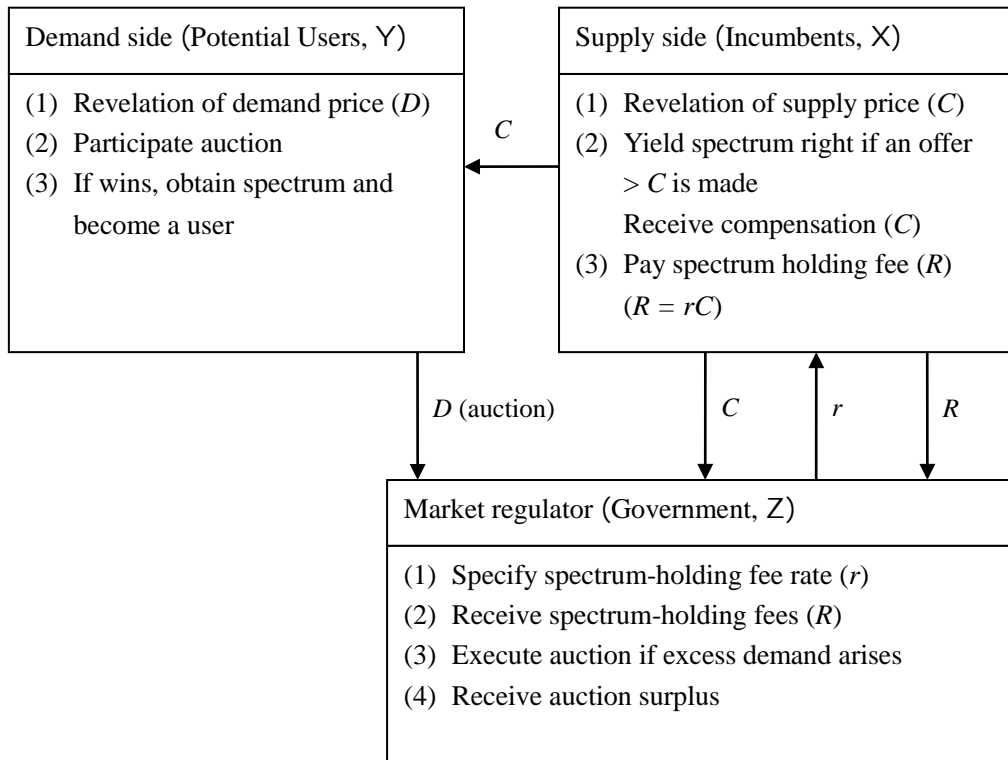
**Figure 4:** Overview of spectrum use with extended spectrum market



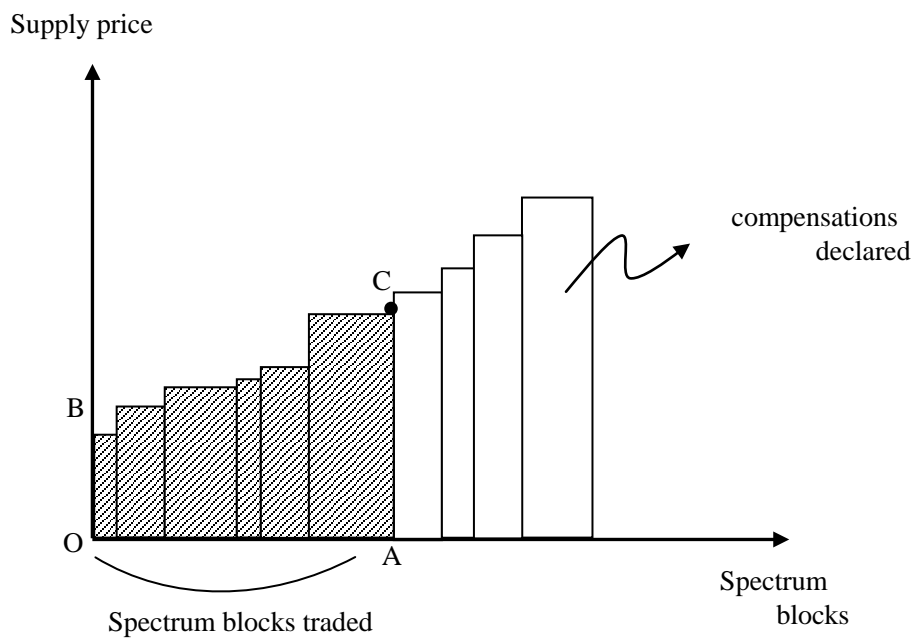
**Figure 4A:** Example of truthful supply prices



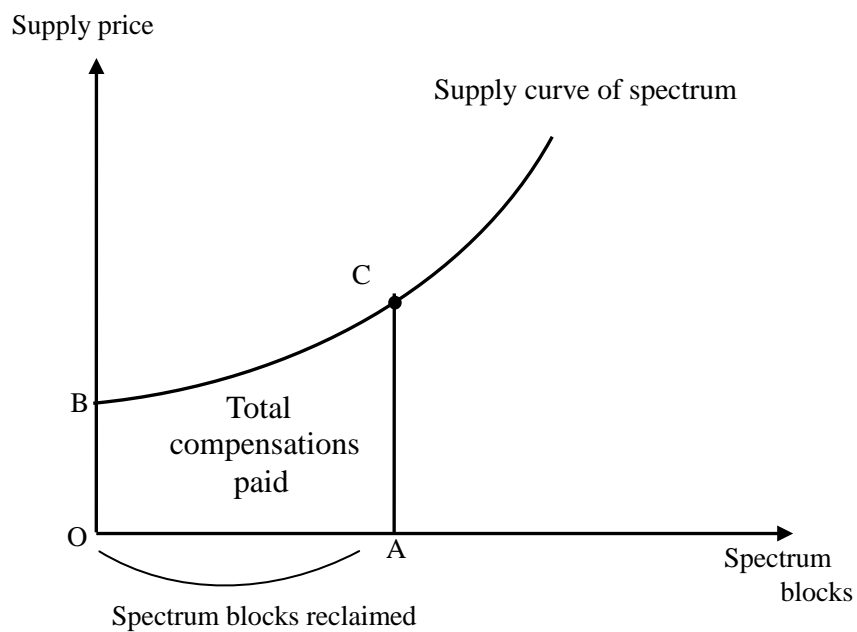
**Figure 4B:** Example of truthful and untruthful supply prices



**Figure 6:** Organization of Extended Spectrum Market



**Figure 7A:** Supply of Spectrum Blocks (1/2)



**Figure 7B:** Supply of Spectrum Blocks (2/2)